

1. Fresnel Double Mirror**(5 pts)**

Hecht, problem 9.20:

In the Fresnel double mirror, $\lambda_0 = 589 \text{ nm}$, the distance of the screen from the two virtual sources (Fig. 9.12) is $s = 2 \text{ m}$, $R = 1 \text{ m}$, and the separation of fringes is $\Delta y = 0.5 \text{ mm}$. Determine the inclination angle α between the mirrors.

From the sketch in Fig. 9.12a, the angle to S_1 and S_2 , as seen from the intersection point of the two mirrors, O , is $\angle(S_1, O, S_2) = 2\alpha$. (This can be shown by drawing the geometry around point O or, more easily, by the following physics argument: If $\alpha = 0$, *i.e.*, both mirrors line up, then $S_1 = S_2$ and thus $\angle(S_1, O, S_2) = 0$; if we then increase the tilt of mirror 2 against mirror 1 by $\Delta\alpha$, then the reflection from mirror 2, and therefore also the direction toward S_2 is rotated by $2\Delta\alpha$ around point O , leading to $\angle(S_1, O, S_2) = 2\Delta\alpha$.)

Thus, $\sin \alpha = a/2R$. On the other hand, $\Delta y \approx s\lambda/a$, so

$$\sin \alpha = \frac{s\lambda}{2\Delta y R}, \text{ or } \alpha = 0.0018 \text{ (in rad)} = 0.067^\circ.$$

2. Fizeau Fringes**(3 pts)**

Hecht, problem 9.34:

A piece of paper ($d = 76.18 \text{ }\mu\text{m}$) separates two glass plates (*e.g.*, microscope slides) at one end while the plates touch at their other end. Determine the no. of fringes seen across the length of the two glass plates.

At the end where the slides touch, there is no optical path difference. At the other end, the OPD Λ is $2d$. The number of fringes distributed along the slides is $N = \text{rounddown}[\Lambda/\lambda_0]$, so

$$N = 2 \cdot 76180 \text{ nm} / (500 \text{ nm}) = 304.$$

3. Index of Air**(4 pts)**

Hecht, problem 9.37:

How many fringe shifts will be seen in a Michelson interferometer in monochromatic light ($\lambda_0 = 600 \text{ nm}$) if a path length of $x = 10 \text{ cm}$ in one of the arms changes from air ($n_{\text{air}} = 1.00029$) to vacuum?

The OPD is $\Lambda = (n_{\text{air}} - 1)x = N(\lambda_0/2)$. Hence, $N = \text{rounddown}[2(n_{\text{air}} - 1)x/\lambda_0] = 96$.

4. Anti-Reflective Coating I**(5 pts)**

Hecht, problem 9.43:

For normal incidence and $n_0 \approx 1$, show that the reflected irradiance of a film-coated interface with a film thickness $d = \lambda_0/4$ is always smaller than the reflected irradiance of the bare interface, as long as $n_0 < n_f < n_1$.

For $\theta_i = 0$, $r = \frac{n_t - n_i}{n_t + n_i}$ and $t = \frac{2n_i}{n_t + n_i}$; for the bare interface, this yields $r = \frac{n_1 - 1}{n_1 + 1}$. The amplitude of

the ray reflected after 2 passes through the film is $r' = t_{0 \rightarrow f} r_{f/1} t_{f \rightarrow 0} = \frac{2}{n_f + 1} \frac{n_1 - n_f}{n_1 + n_f} \frac{2n_f}{1 + n_f}$. Hence for

$n_0 < n_f < n_1$, both $r > 0$ and $r' > 0$ if it were not for the OPD of the ray transmitted through the film, which is $\Delta = 2d = \lambda_0/2$. This OPD adds a phase shift of π , so overall, $r' > 0$ always holds. Thus the total reflection amplitude $\rho = r + r' < r$.

5. Anti-Reflective Coating II**(3 pts)**

Hecht, problem 9.47:

A camera lens ($n_{gl} = 1.55$) is coated with a cryolite film ($n_{cr} = 1.30$) as an antireflective coating. What is the optimal film thickness for incident green light ($\lambda_0 = 500$ nm)?

$$d = \lambda_f/4 = \lambda_0/4n_{cr} = 500 \text{ nm}/5.2 = 96 \text{ nm}.$$

6. Fabry-Perot Interferometer (Bonus Problem)**(5 pts)**

Hecht, problem 9.40:

The mirrors in a Fabry-Perot interferometer have a reflection coefficient $r = 0.8944$. Determine (a) the coefficient of finesse F , (b) the half-width γ , (c) the finesse Φ and (d) the contrast factor, $C = (I_t/I_i)_{\max}/(I_t/I_i)_{\min}$.

$$R = r^2 = 0.8, \text{ hence}$$

$$(a) F = \frac{4R}{(1-R)^2} = 80.$$

$$(b) \gamma = 4 \arcsin(F^{-1/2}) = 0.448 \text{ (in rad)}.$$

$$(c) \Phi = 2\pi/\gamma = 14.$$

$$(d) C = 1 + F = 81.$$